Integrated structure and control design

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1 Abstract

Generally, a controlled system is obtained by designing the structure and the controller independently. This sequential design approach usually yields suboptimal system performance since the structure and the controller might struggle with each other to achieve the required closed-loop behavior. This contribution proposes an algorithm for integrated structure and control design problem to achieve the overall system performance in an optimal sense for instance quantified in terms of H_∞ norm.

2 Overview of the presentation

Generally, a controller is designed to optimize specified measures of closed-loop system transfer functions, e.g. the H_∞ norm of the sensitivity function. For the structural parameter vector $p \in P$, with $\underline{p} \leq p \leq \overline{p}$, the integrated structure and H_∞ control design problem is formulated as follows:

$$\begin{aligned} &\underset{p \in P}{\text{minimize}} \quad \underset{X,K}{\text{minimize}} \quad \gamma \\ &\text{subject to} \quad \begin{bmatrix} XA_{cl} + A_{cl}{}^T X & XB_{cl} & C_{cl}{}^T \\ B_{cl}{}^T X & -\gamma I & D_{cl}{}^T \\ C_{cl} & D_{cl} & -\gamma I \end{bmatrix} \prec 0 \end{aligned} \tag{1}$$

Where, γ is the performance parameter, K is the controller, $A_{cl}(p,K)$, $B_{cl}(p,K)$, $C_{cl}(p,K)$ and $D_{cl}(p,K)$ are the closed-loop system matrices, and X is the Lyapunov matrix. The following two-level algorithm is proposed to solve this problem:

- Lower Level: For the fixed parameter vector p, solve the minimization problem given in Equation (1), i.e.

$$\begin{aligned} &\underset{X,K}{\text{minimize}} \quad \gamma \\ &\text{subject to} \quad \begin{bmatrix} XA_{cl} + A_{cl}{}^T X & XB_{cl} & C_{cl}{}^T \\ B_{cl}{}^T X & -\gamma I & D_{cl}{}^T \\ C_{cl} & D_{cl} & -\gamma I \end{bmatrix} \prec 0 \end{aligned}$$

- Upper Level: Minimize the optimal value of γ

$$\underset{p \in P}{\text{minimize}} \quad \gamma^*$$

The minimization problem defined in the Lower Level is a bilinear matrix inequality (BMI) problem. However, various techniques have been proposed to convert this problem to a convex semi-definite programming problem, for example: see [3]. The minimization problem defined in Upper Level can be solved by using a quasi-Newton gradient based approach using the first order sensitivity $\frac{d\gamma^*}{dp}$ which can be calculated from the Lagrange multiplier of the Lower Level optimization problem, see [2].

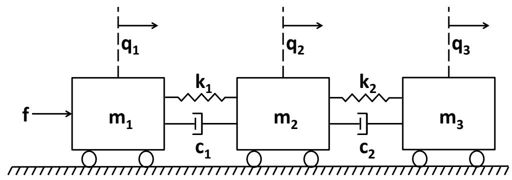

Figure 1: 3 mass-spring-damper system

The numerical example of a three mass-spring-damper system, shown in Figure 1, with m_1 and k_1 as the structural design parameters is used to implement the proposed algorithm. It is required to simultaneously design a colocated contoller which maximizes the bandwidth and limits the residual vibration. It will be shown that the proposed algorithm converges to a local minimum for this particular example.

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